

Assertion Exercises

Question 1 - Concrete Assertions

For each of the following separation-logic assertions, determine whether each assertion is satisfiable or unsatisfiable. When satisfiable, describe the heaps that satisfy the assertion.

- (a) $10 \mapsto 1 \star 10 \mapsto 1$ (d) $10 \mapsto 1 \wedge 11 \mapsto 2$ (g) $10 \mapsto - \star 10 \mapsto 1$
(b) $10 \mapsto 1 \star 10 \mapsto 2$ (e) $10 \mapsto 1 \vee 10 \mapsto 2$ (h) $(10 \mapsto 11 \star 11 \mapsto -) \vee 10 \mapsto 0$
(c) $10 \mapsto 1 \wedge 11 \mapsto 1$ (f) $10 \mapsto - \wedge 10 \mapsto 1$ (i) $(10 \mapsto 1 \star \text{true}) \wedge (11 \mapsto 2 \star \text{true})$

(The notation $10 \mapsto -$ is shorthand for $\exists v. 10 \mapsto v$.)

Answer:

When it comes to heap predicates, think of \star as separation, and of \wedge as overlap.

- (a) Unsatisfiable: \star is separation, so the addresses of the heaps satisfying the left-hand and the right-hand side must be *different*. This is not the case, as they both equal 10; this is called *duplicated resource*. Note that the values do not play a role in determining satisfiability in this case.
- (b) Unsatisfiable, for the same reason as (a).
- (c) Unsatisfiable: \wedge is overlap, so the addresses of the heaps satisfying the left-hand and the right-hand side must be *the same*. This is not the case, as we have 10 on the one side, and 11 on the other. As in (a), the values do not play a role in determining satisfiability in this case.
- (d) Unsatisfiable, for the same reason as (c).
- (e) Satisfiable, by the heaps $\{10 \mapsto 1\}$ and $\{10 \mapsto 2\}$.
- (f) Satisfiable: \wedge is overlap, and we do have that the addresses are the same. Next, we move on to examine the values: on the right-hand side, we have $10 \mapsto 1$, so the question becomes if the assertion on the left-hand side is satisfied by the heap $\{10 \mapsto 1\}$. Since it is, the entire assertion is satisfied by the heap $\{10 \mapsto 1\}$.
- (g) Unsatisfiable, for the same reason as (a).
- (h) Satisfiable, by heaps $\{10 \mapsto 11, 11 \mapsto v\}$, for any v , or by $\{10 \mapsto 0\}$.
- (i) Satisfiable: \wedge is overlap, so the addresses of the heaps satisfying the left-hand and the right-hand side must be the same. The left-hand side is satisfied by any heap that contains $10 \mapsto 1$, and the right-hand side is satisfied by any heap that contains $11 \mapsto 2$. The overlap between these two sets of heaps is the solution: any heap that contains both $10 \mapsto 1$ and $11 \mapsto 2$.

Question 2 - Heap Predicates

For each of the following separation-logic assertions and variable stores, either provide a heap that, together with the given variable store, satisfies the assertion or briefly justify why the assertion is unsatisfiable.

1. assertion $x \mapsto y \star y \mapsto x$ with variable store $\{x \rightarrow 10, y \rightarrow 42\}$
2. assertion $x \mapsto y \wedge y \mapsto x$ with variable store $\{x \rightarrow 10, y \rightarrow 42\}$
3. assertion $x \mapsto y \wedge y \mapsto x$ with variable store $\{x \rightarrow 10, y \rightarrow 10\}$

Answer:

- (a) Satisfiable using the heap $\{10 \mapsto 42, 42 \mapsto 10\}$.
- (b) Unsatisfiable.
- (c) Satisfiable using the heap $\{10 \mapsto 10\}$