

$$\begin{array}{c}
\frac{x \notin \text{pv}(E')}{\Gamma \vdash \{x \doteq E' \star E \in \text{Val}\} \ x := E \ \{x \doteq E[E'/x]\}} \text{assign} \\
\\
\frac{x \notin \text{pv}(E')}{\Gamma \vdash \{x \doteq E' \star E \mapsto E_1\} \ x := [E] \ \{x \doteq E_1[E'/x] \star E[E'/x] \mapsto E_1[E'/x]\}} \text{lookup} \\
\\
\frac{}{\Gamma \vdash \{E_1 \mapsto E \star E_2 \in \text{Val}\} \ [E_1] := E_2 \ \{E_1 \mapsto E_2\}} \text{mutate} \\
\\
\frac{x \notin \text{pv}(E')}{\Gamma \vdash \{x \doteq E' \star E \in \mathbb{N}\} \ x := \text{new}(E) \ \{\exists x. x \doteq x \star \bigotimes_{0 \leq i < E[E'/x]} ((x+i) \mapsto \text{null})\}} \text{new} \\
\\
\frac{}{\Gamma \vdash \{E \mapsto E_1\} \ \text{dispose}(E) \ \{\text{emp}\}} \text{dispose} \quad \frac{}{\Gamma \vdash \{\text{emp}\} \ \text{skip} \ \{\text{emp}\}} \text{skip} \quad \frac{\Gamma \vdash \{P\} \ C_1 \ \{Q\} \quad \Gamma \vdash \{Q\} \ C_2 \ \{R\}}{\Gamma \vdash \{P\} \ C_1; \ C_2 \ \{R\}} \text{seq} \\
\\
\frac{\Gamma \vdash \{P \wedge E\} \ C_1 \ \{Q_1\} \quad \Gamma \vdash \{P \wedge \neg E\} \ C_2 \ \{Q_2\}}{\Gamma \vdash \{P \star E \in \text{Bool}\} \ \text{if } (E) \ C_1 \ \text{else } C_2 \ \{Q_1 \vee Q_2\}} \text{if} \quad \frac{\Gamma \vdash \{P \wedge E\} \ C \ \{P \star E \in \text{Bool}\}}{\Gamma \vdash \{P \star E \in \text{Bool}\} \ \text{while } (E) \ C \ \{P \wedge \neg E\}} \text{while} \\
\\
\frac{\Gamma \vdash \{P\} \ C \ \{Q\} \quad \text{mod}(C) \cap \text{pv}(R) = \emptyset}{\Gamma \vdash \{P \star R\} \ C \ \{Q \star R\}} \text{frame} \quad \frac{\vDash P \Rightarrow P' \quad \Gamma \vdash \{P'\} \ C \ \{Q'\} \quad \vDash Q' \Rightarrow Q}{\Gamma \vdash \{P\} \ C \ \{Q\}} \text{cons} \\
\\
\frac{\Gamma \vdash \{P\} \ C \ \{Q\}}{\Gamma \vdash \{\exists x. P\} \ C \ \{\exists x. Q\}} \text{exists} \quad \frac{\Gamma \vdash \{P_1\} \ C \ \{Q\} \quad \Gamma \vdash \{P_2\} \ C \ \{Q\}}{\Gamma \vdash \{P_1 \vee P_2\} \ C \ \{Q\}} \text{disj} \\
\\
\frac{\{P\} \ f(\vec{x}) \ \{Q\} \in \Gamma}{\Gamma \vdash \{\vec{E} \in \text{Val} \star P[\vec{E}/\vec{x}]\} \ y := f(\vec{E}) \ \{Q[y/\text{ret}]\}} \text{fcall}
\end{array}$$

Figure 6.1: Separation Logic for the WHILE Language